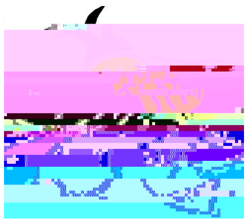
# INTEGRATION BEE ROUND ONE: DEFINITE INTEGRALS

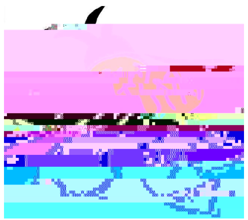


You will have rum kglsrcq to evaluate each of the ffteen defnite integrals that will be displayed one at a time on this screen.

At the end of the two minutes, all hands must go up and judges will grade your answers immediately. If you wish to protest an answer, notify a grader before the next integral is displayed.

For each correct answer, you will receive one rafe ticket to be entered for prizes that will be drawn after dinner.

# INTEGRATION BEE ROUND ONE: DEFINITE INTEGRALS



- You do lmr need to combine multiple logs or fractions with irrational numbers (e.g. $\pi$, e, $\ln 3$).

  $2\ln 3$ ✓, $\ln 9$ ✓ $\frac{1}{e} + 2$ ✓, $\frac{1+2e}{e}$ ✓

- However, lm negative powers shou t te

# READY,

# GET SET,...

# 2:00

INTEGRAL #1

$$\int_1^2 \frac{20x+24}{x^3}\,dx$$

2:00

## INTEGRAL #1

$$\int_1^2 \frac{20x + 24}{x^3}\, dx$$

$$= \int_1^2 20x^{2} + 24x^{3}\, dx$$

$$= \left[ \frac{20}{x} + \frac{12}{x^2} \right]^2$$

# READY,

# GET SET,...

# 2:00

INTEGRAL #2

$$\int_{1}^{4} \frac{6}{\sqrt{x}\,(1+\sqrt{x})^{3}}\,dx$$

2:00

# INTEGRAL #2

$$\int_1^4 \frac{6}{\sqrt{x}(1+\sqrt{x})^3}\,dx$$

$$= 12\int_2^3 u^{-3}\,du \qquad u = 1+\sqrt{x}, \quad du = \frac{1}{2\sqrt{x}}\,dx$$

$$= -6\,\Big[u^{-2}\Big]_2^3$$

$$= -6\left(\frac{1}{9}-\frac{1}{4}\right)$$

$$= \boxed{\frac{5}{6}}$$

INTEGRAL #3

$$\int_1^2 e^{3\ln x^2 \quad 2\ln x}\, dx$$

2:00

## INTEGRAL #3

$$\int_1^2 e^{3\ln x^2 - 2\ln x}\,dx$$

$$= \int_1^2 e^{\ln x^6 - \ln x^2}\,dx$$

$$= \int_1^2 e^{\ln x^4}\,dx$$

$$= \int_1^2 x^4\,dx$$

$$= \left[\frac{x^5}{5}\right]_1^2 = \frac{1}{5}\left(2^5 - 1\right) = \boxed{\frac{31}{5} \text{ or } 6\frac{1}{5}}$$

# READY,

# GET SET,...

2:00

2:00

# READY,

# GET SET,...

# 2:00

$$\int_{0}^{3} \frac{2x \quad 3}{x^{2} \quad 3x \quad 18}\, dx$$

2:00

## INTEGRAL #5

$$\int_{0}^{3} \frac{2x \quad 3}{x^2 \quad 3x \quad 18}\, dx$$

$$= \int_{18}^{18} \frac{1}{u}\, du \qquad u = x^2 \quad 3x \quad 18, \quad du = (2x \quad 3)\, dx$$

$$=$$

INTEGRAL #6

READY,

GET SET,...

2:00

INTEGRAL #6

$$\int_{1}^{e^4} \frac{(\ln x)^2}{x}\,dx$$

2:00

INTEGRAL # 6

$$\int_1^{e^4} \frac{(\ln x)^2}{x}\,dx$$

$$= \int_0^4 u^2\,du \qquad u = \ln x, \quad du = \frac{1}{x}\,dx$$

$$= \left.\frac{u^3}{3}\right|_0^4$$

$$= \boxed{\frac{64}{3} \text{ or } 21\frac{1}{3}}$$

INTEGRAL #7

$$\int_{1}^{1} 18x^5 + 3x^2 + x \, dx$$

2:00

## INTEGRAL #7

$$\int_{-1}^{1} 18x^5 + 3x^2 + x \, dx$$

$$= \left[ 3x^6 + x^3 + \frac{x^2}{2} \right]_{-1}^{1}$$

$$= 3 + 1 + \frac{1}{2} - \left( 3 - 1 + \frac{1}{2} \right)$$

$$= \boxed{2}$$

INTEGRAL #8

$$\int_0^{\pi/6} \sec 2x \tan 2x \, dx$$

2:00

# INTEGRAL #8

$$\int_0^{\pi/6} \sec 2x \tan 2x \, dx$$

$$= \frac{1}{2} \int_0^{\pi/3} \sec u \tan u \, du \qquad u = 2x, \quad du = 2\,dx$$

$$= \frac{1}{2} \Big[ \sec u \Big]_0^{\pi/3}$$

$$= \frac{1}{2}(2 - 1)$$

$$= \boxed{\frac{1}{2}}$$

# READY,

# GET SET,...

2:00

INTEGRAL #9

$$\int_1^1 \frac{x^2+1}{x^3+2x^2+x+2}\,dx$$

2:00

# READY,

# GET SET,...

# 2:00

2:00

INTEGRAL #11

$$\int_0^{\pi} (\sin x + \cos x)^2 \, dx$$

2:00

$$\int_{0}^{13} \frac{1}{\sqrt[3]{(1+2x)^2}}\,dx$$

2:00

## INTEGRAL #12

$$\int_0^{13} \frac{1}{\sqrt[3]{(1+2x)^2}}\,dx$$

$$= \frac{1}{2}\int_1^{27} \frac{1}{\sqrt[3]{u^2}}\,du \qquad u = 1+2x, \quad du = 2\,dx$$

$$= \frac{1}{2}\int_1^{27} u^{-2/3}\,du$$

$$= \left[\frac{1}{2}\,3u^{1/3}\right]_1^{27} = \frac{3}{2}(3-1) = \boxed{3}$$

# READY,

# GET SET,...

# 2:00

[redacted]

2:00

# READY,

# GET SET,...

2:00

$$\int_0^{\pi/2} \sin x \sqrt{1+3\cos x}\,dx$$

2:00

## INTEGRAL #14

$$\int_0^{\pi/2} \sin x \sqrt{1+3\cos x}\,dx$$

$$= -\frac{1}{3}\int_4^1 \sqrt{u}\,du \qquad u = 1+3\cos x,\quad du = -3\sin x\,dx$$

$$= \frac{1}{3}\int_1^4 \sqrt{u}\,du$$

$$= \left[\frac{1}{3}\cdot\frac{2}{3}u^{3/2}\right]_1^4$$

$$= \frac{2}{9}(8-1) = \boxed{\frac{14}{9} \text{ or } 1\frac{5}{9}}$$

# READY,

# GET SET,...

# 2:00

$$\int_{2}^{3} \frac{x+5}{(4-x)^4}\,dx$$

2:00

# INTEGRAL #15

$$\int_2^3 \frac{x+5}{(4-x)^4}\,dx$$

$$= -\int_2^1 \frac{4-u+5}{u^4}\,du \qquad u = 4-x,\; x = 4-u,\; du = -dx$$

$$= \int_2^1 u^{-3} - 9u^{-4}\,du$$

$$= \left[\frac{3}{u^3} - \frac{1}{2u^2}\right]_2^1$$

$$= \left[\frac{5}{2} - \frac{1}{4}\right] = \boxed{\frac{9}{4} \text{ or } 2\frac{1}{4}}$$